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Research Article

On the Derivatives of Bernstein Polynomials: An Application for the Solution of High Even-Order Differential Equations

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A new formula expressing explicitly the derivatives of Bernstein polynomials of any degree and for any order in terms of Bernstein polynomials themselves is proved, and a formula expressing the Bernstein coefficients of the general-order derivative of a differentiable function in terms of its Bernstein coefficients is deduced. An application of how to use Bernstein polynomials for solving high even-order differential equations by Bernstein Galerkin and Bernstein Petrov-Galerkin methods is described. These two methods are then tested on examples and compared with other methods. It is shown that the presented methods yield better results.

1. Introduction

Bernstein polynomials [1] have many useful properties, such as, the positivity, the continuity, and unity partition of the basis set over the interval $[0, 1]$. The Bernstein polynomial bases vanish except the first polynomial at $x = 0$, which is equal to 1 and the last polynomial at $x = 1$, which is also equal to 1 over the interval $[0, 1]$. This provides greater flexibility in imposing boundary conditions at the end points of the interval. The moments x^m is nothing but Bernstein polynomial itself. With the advent of computer graphics, Bernstein polynomial restricted to the interval $x \in [0, 1]$ becomes important in the form of Bezier curves [2]. Many properties of the Bézier curves and surfaces come from the properties of the Bernstein