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CLASSIFICATION OF RINGS SATISFYING SOME CONSTRAINTS ON SUBSETS

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Abstract. Let R be an associative ring with identity 1 and J(R) the Jacobson radical of R. Suppose that m _ 1 is a fixed positive integer and R an m-torsion-free ring with 1. In the present paper, it is shown that R is commutative if R satisfies both the conditions (i) $[x_m, y_m] = 0$ for all x, y 2 R\J(R) and (ii) $[x, [x, y_m]] = 0$, for all x, y 2 R\J(R). This result is also valid if (ii) is replaced by (ii)' $[(yx)_m x_m - x_m(xy)_m, x] = 0$, for all x, y 2 R\N(R). Our results generalize many well-known commutativity theorems (cf. [1], [2], [3], [4], [5], [6], [9], [10], [11] and [14]).

1. Introduction

Throughout, R represents an associative ring with identity 1, Z(R) the centre of R,U(R) denotes the group of units of R, J(R) the Jacobson radical of R,N(R) the set of nilpotent elements of R, and C(R) the commutator ideal of R. As usual, for any $x, y \in R$, the symbol $[x, y]$ will stand for the commutator $xy - yx$. Let $m \geq 1$ be a fixed positive integer and a non-empty subset S of R. We consider the following ring properties.

$C_1(m, S)$ $[x_m, y_m] = 0$ for all $x, y \in S$.
$C_2(m, S)$ $[x, [x, y_m]] = 0$ for all $x, y \in S$.
$C_3(m, S)$ $(xy)_m = x_m y_m$ for all $x, y \in S$.
$C_4(m, S)$ $(xy)_m - x_m y_m \in Z(R)$ for all $x, y \in S$.
$C_5(m, S)$ $(xy)_m - y_m x_m \in Z(R)$ for all $x, y \in S$.
$C_6(m, S)$ $[(xy)_m$ پ} $y_m x_m, x] = 0 = [(yx)_m$ پ} $x_m y_m, x]$ for all $x, y \in S$.
$C_7(m, S)$ $[(yx)_m x_m - x_m(xy)_m, x] = 0$ for all $x, y \in S$.
$Q(m)$ For any $x, y \in R$, $m[x, y] = 0$ implies $[x, y] = 0$.

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